

Topic E: The quadratic formula



You can solve a quadratic equation using the **quadratic formula**. The quadratic formula can also be used to quickly determine how many roots a quadratic equation has.

Key point

The quadratic formula for $ax^2 + bx + c = 0$ is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example 1

Solve the equation $3x^2 - 5x - 7 = 0$ using the quadratic formula.

$$a = 3, b = -5, c = -7$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 3 \times (-7)}}{2 \times 3}$$

$$= \frac{5 \pm \sqrt{109}}{6}$$

$$= 2.57 \text{ or } -0.91 \text{ (to 2 dp)}$$

Substitute into the formula, taking care with negatives.

Use your calculator to give answer as a decimal:

$$\frac{5 + \sqrt{109}}{6} = 2.57 \text{ and}$$

$$\frac{5 - \sqrt{109}}{6} = -0.91$$

You can also use the equation solver on your calculator to solve quadratic equations.



Use the quadratic formula to solve the quadratic equation $7x^2 - 4x - 6 = 0$

Try It 1

Inside the square root of the quadratic formula you have the expression $b^2 - 4ac$. This expression is called the **discriminant**. You can use the discriminant to determine how many roots the equation has.



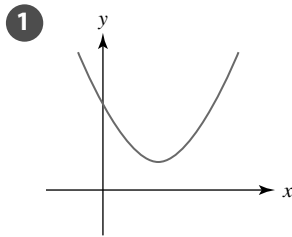
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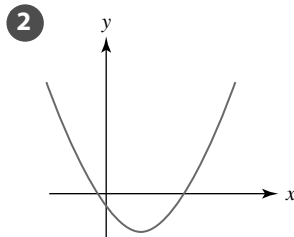


- 1 If $b^2 - 4ac < 0$ then the equation has no real roots.
- 2 If $b^2 - 4ac > 0$ then the equation has two real roots.
- 3 If $b^2 - 4ac = 0$ then the equation has one real root.

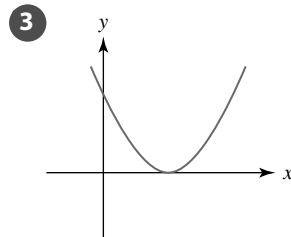
Key point



The curve does not cross the x -axis so the discriminant is negative.



The curve crosses the x -axis twice so the discriminant is positive.



The curve touches the x -axis once so the discriminant equals zero.

Example 2

Given that the quadratic equation $x^2 + 3x + k + 1 = 0$ has exactly one solution, find the value of k

$$a = 1, b = 3, c = k + 1$$

$$\text{So } b^2 - 4ac = 3^2 - 4 \times 1 \times (k + 1)$$

$$= 5 - 4k$$

$$5 - 4k = 0 \Rightarrow k = \frac{5}{4}$$

Find the discriminant.

The equation has exactly one solution so the discriminant is zero.

Given that the quadratic equation $kx^2 - x + 5 = 0$ has exactly one solution, find the value of k

Try It 2



Example 3

Given that the quadratic equation $5x^2 + 3x - k = 0$ has real solutions, find the range of possible values of k

$$a = 5, b = 3, c = -k$$

$$\text{So } b^2 - 4ac = 3^2 - 4 \times 5 \times (-k)$$

$$= 25 + 20k$$

$$25 + 20k \geq 0 \Rightarrow k \geq -\frac{5}{4}$$

Find the discriminant.

The equation has real solutions so the discriminant is greater than or equal to zero.

Given that the quadratic equation $x^2 + 3x - k = 0$ has real solutions, find the range of possible values of k

Try It 3

Example 4

Given that the quadratic equation $-x^2 + 7x + 3 - k = 0$ has no real solutions, find the range of possible values of k

$$a = -1, b = 7, c = 3 - k$$

$$\text{So } b^2 - 4ac = 7^2 - 4 \times (-1) \times (3 - k)$$

$$= 61 - 4k$$

$$61 - 4k < 0 \Rightarrow k > \frac{61}{4}$$

Find the discriminant.

The equation has no solutions so the discriminant is negative.

Given that the quadratic equation $kx^2 - 7x + 1 = 0$ has no real solutions, find the range of possible values of k

Try It 4



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1 Use the quadratic formula to solve each of these equations.

a $7x^2 + 3x - 8 = 0$

b $-x^2 + 4x - 2 = 0$

c $x^2 - 12x + 4 = 0$

2 Work out how many real solutions each of these quadratic equations has.

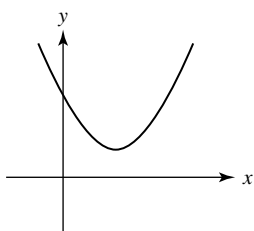
a $x^2 - 5x + 7 = 0$

b $7 - 2x - 3x^2 = 0$

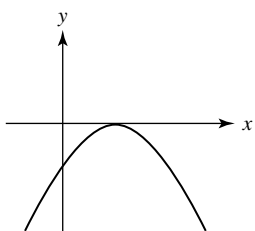
c $4x^2 - 28x + 49 = 0$

3 Choose a possible equation from the box for each of the graphs.

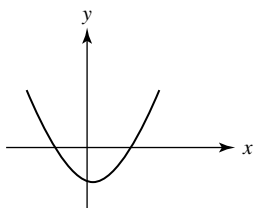
a



b



c



$$y = -4x^2 + 12x - 9$$

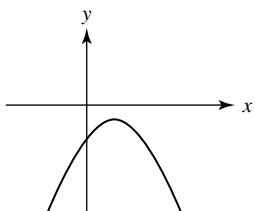
$$y = -x^2 + 2x - 4$$

$$y = 7x^2 - 5x + 4$$

$$y = -x^2 + x + 6$$

$$y = 6x^2 - x - 15$$

d



4 Find the value of k in each equation given that they each have exactly one solution.

a $3x^2 + 2x - k = 0$

b $kx^2 - x + 4 = 0$

c $2x^2 + 5x + k - 5 = 0$

5 Find the range of possible values of k for each equation given that they all have real solutions.

a $x^2 + 3x - 3k = 0$

b $kx^2 - 7x + 4 = 0$

c $-x^2 + 6x - k - 2 = 0$

6 Find the range of possible values of k for each equation given that they all have no real solutions.

a $5x^2 - x + 2k = 0$

b $-kx^2 + 4x + 5 = 0$

c $6x^2 - 5x + 3 - 2k = 0$
