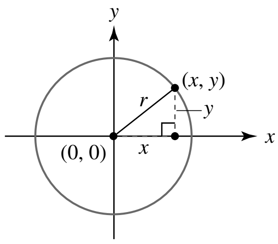
1 Bridging Unit: Algebra 1

Topic G: Circles

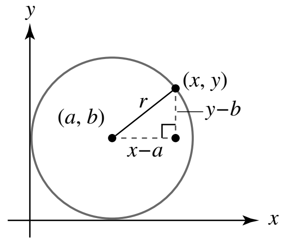
Bridging to Ch1.5

You can use the centre and radius of a circle to define its equation, and to define the equation of a tangent to circle at a given point. You can also find points of intersection between a circle and a line or chord.



Using Pythagoras' theorem, a circle of radius r, with centre at the origin, has equation $x^2+y^2=r^2$

Following a similar method, you can write down the equation of a circle with centre (a, b) and radius r, using a general point (x, y) on the circle, as shown in the diagram.



The horizontal distance between the centre (a, b) and the point on the circle (x, y) is the difference between the x-coordinates. The vertical distance between the centre (a, b) and the point on the circle (x, y) is the difference between the y-coordinates.

Using Pythagoras' theorem: $r^2=(x-a)^2+(y-b)^2$

Key point

A circle of radius r and centre (a, b) has equation $(x-a)^2+(y-b)^2=r^2$

Example 1

a Find the centre and radius of the circle with equation $(x-5)^2+(y+1)^2=9$

b Write the equation of a circle with centre (–3, 7) and radius 4

a The centre is at $(5,-1)$

Equation is $(x-5)^2+(y-(-1))^2=9$ so $a=5$ and $b=-1$

The radius is $\sqrt{9}=3$

Remember to find the positive square root.

b $a=-3$, $b=7$ and $r=4$

So equation is $(x+3)^2+(y-7)^2=16$

Remember to square the radius.

Try It 1

a Find the centre and radius of the circle with equation $(x+2)^2+(y-8)^2=25$

b Write the equation of a circle with centre (7, –9) and radius 8

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If you have the equation of a circle in expanded form then you can complete the square, as shown in Topic D, to write it in the form $(x-a)^2+(y-b)^2=r^2$ which will enable you to state the centre and radius.

Example 2

Find the centre and radius of the circle with equation $x^2+y^2-8x+4y+2=0$

$x^2-8x+y^2+4y+2=0$ — Group the terms involving x and the terms involving y

$(x-4)^2-16+(y+2)^2-4+2=0$ — Complete the square for x^2-8x and y^2+4y

$(x-4)^2+(y+2)^2=18$

So the centre is $(4, -2)$ and the radius is $\sqrt{18}=3\sqrt{2}$

Try It 2

Find the centre and radius of the circles with these equations.

a $x^2+y^2-10y+16=0$ **b** $x^2+y^2+6x-12y=0$

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You can use a diameter of a circle to find the equation of the circle.

Key point

If AB is the diameter of a circle then

- the centre of the circle is the midpoint of AB
- the radius of the circle is half the length of the diameter AB

Example 3

Find the equation of the circle with diameter AB where A is (3, –8) and B is (–5, 4)

Centre is $\left(\frac{3+(-5)}{2}, \frac{(-8)+4}{2}\right)$

$= (-1, -2)$

The centre is the midpoint of AB. Use $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Radius is $\frac{1}{2}\sqrt{(-5-3)^2+(4-(-8))^2}$

$= \frac{1}{2}\sqrt{(-8)^2+(12)^2}$

$= 2\sqrt{13}$

The radius is half of the length of AB

So the equation of the circle is $(x+1)^2+(y+2)^2=52$

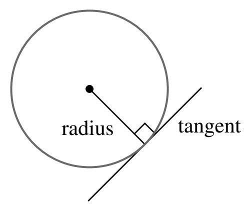
Use $(x-a)^2+(y-b)^2=r^2$ and remember to square the radius: $(2\sqrt{13})^2=52$

Try It 3

Find the equation of the circle with diameter AB where A is (4, 6) and B is (2, –4)

A **tangent** to a circle is a line which is perpendicular to a radius of the circle. Note that a tangent will intersect a circle exactly once.

You can use these facts to find the equation of a tangent to a circle.



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Example 4

A circle has equation $(x+3)^2+(y-7)^2=26$

a Show that the point (–4, 2) lies on the circle.

b Find the equation of the tangent to the circle that passes through the point (–4, 2)

a $(-4+3)^2+(2-7)^2=(-1)^2+(-5)^2$

$=1+25$

$=26$ so $(-4, 2)$ lies on the circle.

Substitute $x=-4$, $y=2$ into the equation.

b Centre of circle is $(-3, 7)$

Gradient of radius is $\frac{2-7}{-4-(-3)}=\frac{-5}{-1}=5$

Use $m=\left(\frac{y_2-y_1}{x_2-x_1}\right)$

A tangent is perpendicular to a radius so gradient of tangent is $-\frac{1}{5}$

Since $\left(-\frac{1}{5}\right)\times 5=-1$

Therefore equation of tangent is $y-2=-\frac{1}{5}(x+4)$

Use $y-y_1=m(x-x_1)$ with $(x_1, y_1)=(-4, 2)$

Try It 4

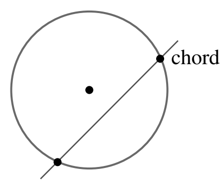
A circle has equation $(x-1)^2+(y+4)^2=50$

a Show that the point (6, 1) lies on the circle.

b Find the equation of the tangent to the circle that passes through the point (6, 1)

You can find the point of intersection of a line and a circle by solving their equations simultaneously. You will need to use the **substitution** method of solving simultaneous equations.

If the line intersects the circle twice then it is a **chord**.



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Example 5

The line $x+3y=12$ and the circle $(x+3)^2+(y-7)^2=4$ intersect at the points A and B

a Find the coordinates of A and B

b Calculate the length of the chord AB

a $x=12-3y$

$(12-3y+3)^2+(y-7)^2=4$

$\Rightarrow (15-3y)^2+(y-7)^2=4$

$\Rightarrow 225-90y+9y^2+y^2-14y+49=4$

$\Rightarrow 10y^2-104y+270=0$

$\Rightarrow y=5.4$ or $y=5$

$x=12-3(5.4) \Rightarrow x=-4.2$

$x=12-3(5)=-3$

So they intersect at $A(-4.2, 5.4)$ and $B(-3, 5)$

Rearrange the equation of the line to make either x or y the subject (whichever is easiest).

Substitute for x (or y) in the equation of the circle.

Simplify, then use the equation solver on your calculator.

Substitute the values of y into the rearranged equation of the line to find the values of x

The line and the circle will intersect twice unless the line is a **tangent** to the circle.

b Length of chord $AB=\sqrt{(-3-(-4.2))^2+(5-5.4)^2}$

$=\sqrt{1.2^2+(-0.4)^2}$

$=\frac{2}{5}\sqrt{10}$ (= 1.26 to 3 significant figures)

Use $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

You can find points of intersection using a graphic calculator.

Try It 5

The line $3x+y=5$ intersects the circle $x^2+(y-4)^2=17$ at the points A and B

a Find the coordinates of A and B

b Calculate the length of the chord AB

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Example 6

Show that $x-y=12$ is a tangent to the circle $(x-6)^2+(y+2)^2=8$

$y=x-12$ — Rearrange the equation of the line to make either x or y the subject.

$(x-6)^2+(x-12+2)^2=8$ — Substitute for y (or x) into the equation of the circle.

$\Rightarrow (x-6)^2+(x-10)^2=8$

$\Rightarrow x^2-12x+36+x^2-20x+100=8$ — Expand the brackets.

$\Rightarrow 2x^2-32x+128=0$ — Simplify.

$b^2-4ac=(-32)^2-4\times2\times128=0$ — If the discriminant is zero then there is exactly one solution.

So they meet once only.

Hence $x-y=12$ is a tangent to $(x-6)^2+(y+2)^2=8$

To show that a line is a tangent to a circle you can show that they only intersect once.

Try It 6

Show that $2x-y+11=0$ is a tangent to the circle $(x-5)^2+(y-1)^2=80$

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Bridging Exercise Topic G

Bridging to Ch1.5

1 Write the equations of these circles.

a circle with radius 7 and centre (2, 5)

b circle with radius 4 and centre (–1, –3)

c circle with radius $\sqrt{2}$ and centre (–3, 0)

d circle with radius $\sqrt{5}$ and centre (4, –2)

2 Find the centre and the radius of the circles with these equations.

a $(x-5)^2+(y-3)^2=16$

b $(x+3)^2+(y-4)^2=36$

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c $(x-9)^2+(y+2)^2=100$

d $(x+3)^2+(y+1)^2=80$

e $(x-\sqrt{2})^2+(y+2\sqrt{2})^2=32$

f $\left(x+\frac{1}{4}\right)^2+\left(y+\frac{1}{3}\right)^2=\frac{25}{4}$

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3 Find the centre and the radius of the circles with these equations.

a $x^2+2x+y^2=24$

b $x^2+y^2+12y=13$

c $x^2+y^2-4x+3=0$

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d $x^2+y^2+6x+8y+2=0$

e $x^2+y^2-8x-10y=3$

f $x^2+y^2+14x-2y=5$

g $x^2+y^2+5x-4y+3=0$

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h $x^2 + y^2 - 3x - 9y = 2$

i $x^2 + y^2 - x + 7y + 12 = 0$

4 Find the equation of the circle with diameter AB where the coordinates of A and B are

a $(3, 5)$ and $(1, 7)$

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b $(4, -1)$ and $(2, -5)$

c $(1, -3)$ and $(-9, -6)$

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d $(-3, -7)$ and $(8, -16)$

e $(\sqrt{2}, 4)$ and $(-\sqrt{2}, 6)$

f $(4\sqrt{3}, -\sqrt{3})$ and $(-2\sqrt{3}, -5\sqrt{3})$

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5 Determine whether each of these points lies on the circle with equation $(x-3)^2+(y+2)^2=5$

a $(5, 3)$

b $(1, -1)$

c $(4, 3)$

d $(2, 0)$

6 Determine which of these circles the point $(-3, 2)$ lies on.

a $(x-5)^2+y^2=68$

b $(x+2)^2+(y+1)^2=8$

c $(x-6)^2+(y-2)^2=81$

7 A circle has equation $(x-1)^2+(y+1)^2=10$. Find the equation of the tangent to the circle through the point (2, –4). Write your answer in the form $ax+by+c=0$ where a, b and c are integers.

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8 A circle has equation $(x+3)^2+(y+7)^2=34$. Find the equation of the tangent to the circle through the point (0, –2). Write your answer in the form $ax+by+c=0$ where *a*, *b* and *c* are integers.

9 A circle has equation $x^2+(y-8)^2=153$. Find the equation of the tangent to the circle through the point (3, –4). Write your answer in the form $y=mx+c$

10 A circle has equation $(x+4)^2+y^2=20.5$. Find the equation of the tangent to the circle through the point (0.5, –0.5). Write your answer in the form $y=mx+c$

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11 Find the points of intersection, A and B, between these pairs of lines and circles.

a $x + y = 5, \quad x^2 + y^2 = 53$

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b $y+1=0, \quad (x-1)^2+(y+2)^2=17$

c $2x-y+7=0, \quad (x-2)^2+(y+1)^2=36$

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d $y = 2x + 1, \ (x+4)^2 + (y+6)^2 = 10$

12 The line $3x - 9y = 6$ intersects the circle $(x+7)^2 + (y+3)^2 = 10$ at the points A and B

a Find the coordinates of A and B

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b Calculate the length of the chord AB

13 The line $2x+4y=10$ intersects the circle $(x+5)^2+(y-2)^2=20$ at the points A and B

a Find the coordinates of A and B

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b Calculate the length of the chord AB

14 Show that the line $y = x - 3$ is a tangent to the circle $(x-3)^2 + (y+2)^2 = 2$

15 Show that the line $4x + y = 34$ is a tangent to the circle $(x+1)^2 + (y-4)^2 = 68$

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16 Show that the line $x+3y=25$ is a tangent to the circle $x^2+(y-5)^2=10$

17 Show that the line $y=2x+3$ does not intersect the circle $(x-1)^2+(y+4)^2=1$

18 Show that the line $3x+4y+2=0$ does not intersect the circle $(x+3)^2+(y-6)^2=9$

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