

Topic D: Completing the square



Some quadratics are **perfect squares** such as $x^2 - 8x + 16$ which can be written $(x - 4)^2$. For other quadratics you can **complete the square**. This means write the quadratic in the form $(x + q)^2 + r$

The completed square form of $x^2 + bx + c$ is $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$

Key point

If you have an expression of the form $ax^2 + bx + c$ then first factor out the a , as shown in Example 1

Example 1

Write each of these quadratics in the form $p(x + q)^2 + r$ where p, q and r are constants to be found.

a $x^2 + 6x + 7$ **b** $-2x^2 + 12x$

$$\begin{aligned} \text{a } x^2 + 6x + 7 &= \left(x + \frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 7 \\ &= (x + 3)^2 - 9 + 7 \\ &= (x + 3)^2 - 2 \end{aligned}$$

$$\begin{aligned} \text{b } -2x^2 + 12x &= -2[x^2 - 6x] \\ &= -2[(x - 3)^2 - 9] \\ &= -2(x - 3)^2 + 18 \end{aligned}$$

The constant term in the bracket will be half of the coefficient of x

First factor out the coefficient of x^2 then complete the square for the expression in the square brackets.

Write each of these quadratics in the form $p(x + q)^2 + r$

a $x^2 + 22x$ **b** $2x^2 - 8x - 6$ **c** $-x^2 + 10x$

Try It 1



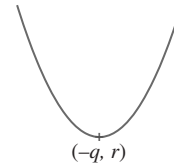
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The turning point on the curve with equation $y = p(x+q)^2 + r$ has coordinates $(-q, r)$, this will be a minimum if p is positive and a maximum if p is negative.

Key point



Example 2

Find the coordinates of the turning point of the curve with equation $y = -x^2 + 5x - 2$

$$\begin{aligned} -x^2 + 5x - 2 &= -\left[x^2 - 5x + 2\right] \\ &= -\left[\left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + 2\right] \\ &= -\left[\left(x - \frac{5}{2}\right)^2 - \frac{17}{4}\right] \\ &= -\left(x - \frac{5}{2}\right)^2 + \frac{17}{4} \end{aligned}$$

So the maximum point is at $\left(\frac{5}{2}, \frac{17}{4}\right)$

First factor out the -1 then complete the square for the expression in the square brackets.

The curve is at its highest point when the bracket is equal to zero: $x - \frac{5}{2} = 0 \Rightarrow x = \frac{5}{2}$

Find the coordinates of the turning point of each of these curves and state whether they are a maximum or a minimum.

Try It 2

- a** $y = x^2 - 3x + 1$ **b** $y = -x^2 - 7x - 12$ **c** $y = 2x^2 + 4x - 1$



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1 Write each of these quadratic expressions in the form $p(x+q)^2+r$

a x^2+8x

b x^2-18x

c x^2+6x+3

d $x^2+12x-5$

e $x^2-7x+10$

f x^2+5x+9

g $2x^2+8x+4$

h $3x^2 + 18x - 6$

i $2x^2 - 10x + 3$

j $-x^2 + 12x - 1$

k $-x^2 + 9x - 3$

l $-2x^2 + 5x - 1$

- 2** Use completing the square to find the turning point of each of these curves and state whether it is a maximum or a minimum.

a $y = x^2 + 14x$

b $y = x^2 - 18x + 3$

c $y = x^2 - 9x$

d $y = -x^2 + 4x$

e $y = x^2 + 11x + 30$

f $y = -x^2 + 6x - 7$ _____

g $y = 2x^2 + 16x - 5$ _____

h $y = -3x^2 + 15x - 2$ _____
